

# CONCEPTUALISING REDUCTION, EMERGENCE AND SELF-ORGANISATION IN COMPLEX DYNAMICAL SYSTEMS

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## 1 SETTING THE SCENE<sup>1</sup>

Fortunately for science, algorithmically compressed representation based on meso-accessible data goes some distance in this cosmos: diverse and complex data can often be shown to express the same, often relatively simple, law and diverse phenomena are often underlain by the same set of laws. Clouds, snow, ice and rain, for instance, all turn out to be re-arrangements of water molecules, just as Democritus had hoped, governed by their common molecular properties and the basic laws of physics. Science provides algorithmically compressed representation whenever it is able to organise information under laws: a single set of initial conditions added to a single law statement in equation form (e.g. Newton's 2<sup>nd</sup> law) suffices to encapsulate an entire dynamics. A final compression occurs whenever the solution of the characterising dynamical equations has a closed-form analytic representation in terms of known mathematical functions, such as form the core mathematical dynamics of simple systems.<sup>2</sup>

Maximising the reach of explicit compression sums up the drive supporting the orthodox vision of the world as formal machine and scientific method as logical algorithm for the construction of its deepest compressed description. Physics is inherently universal: it is the study of those laws of nature (if any) that hold for all physical states. Over the 300+ years since Newton, this study has proven remarkably successful, until now we speak hopefully of including even the universe itself under 'physical states' and obtaining a Theory of Everything. On that basis scientists understandably erected an ideal: show the physics algorithmic compression

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<sup>1</sup>Key terms used here will be explained in the sequel.

<sup>2</sup>McAllister [2003] points out that, since actual empirical data will in general contain noise as well as systematic information, it cannot in general be compressed. Thus the compressible data on which science runs has to be actual data with noise removed (along with any exogenous systematic biases). Of course, achieving such cleaned up data can prove difficult — it is part of what the theory of errors in scientific method is about (cf. [Farrell and Hooker 2009]); but this is an epistemic issue and it is set aside here. It turns out that scientific method is much more complex (and interesting!) than the neat logical algorithms to which the philosophers had hoped to reduce it. Instead it requires many artful decisions if it is to be executed powerfully (cf. [Farrell and Hooker, 2007a; 2007b; Hooker, 1995]. To reduce it to algorithms is at least to reduce all judgement, including creativity, to algorithms, a far-future prospect at best.

universal, that is, reduce all other sciences to applications of the laws of physics. One form of this vision is already clear in the Greek atomist tradition where complexity is constrained to spatio-temporal arrangements; another equally venerable form is found in the Greek plenum, later field, theory tradition where complexity also involves dynamical creation and annihilation.<sup>3</sup>

To that vision philosophy of science has contributed a criterion for its realization, crafted with the logical tools whose use it takes to be obligatory: to reduce X (laws, phenomena in some other science) to physics, show how to deduce X from the laws of physics plus any required initial and constraint ('boundary') conditions, that is, show how all the other sciences are applied physics.<sup>4</sup> Within this perspective reduction represents increased compression since hitherto independent laws are now represented as deductive consequences of their reducing laws. Conversely, emergence represents a constraint on compression. It is helpful to approach emergence and reduction from this perspective because it ultimately makes it easier to recognise the importance of the shift to dynamical criteria for emergence and reduction, permitting these phenomena to be characterised independently of such formal issues.

This orthodox vision has to date carried us so remarkably far in knowing our cosmos as to seem an incredibly unlikely miracle for such finite, fallible creatures as us. It is true that we have played our part in this, massively enlarging our observational niche through invention of instrumental prostheses and massively increasing our logical machinery through the invention of mathematics and computation. But we have still needed the enormous advantage of a penetrable world to work with.<sup>5</sup> Moreover, note immediately this limitation to the maximally explicit version of the vision: the dynamics of many complex systems have no analytic representations and so cannot be given explicit analytic compressed representations, their detail can at best be represented extensionally by computational simulation. And in fact every era since Newton has worried about what might be the principled limits of the compression programme. The last century was consumed by trying to understand the implications of quantum mechanics — itself a triumph of the compression programme — for limits on expanding our reach any further down in scale (cf. note 3).

The emergence of complex systems presents a new challenge to unlimited compression, this time in the form of dynamical emergence and condition-dependent laws. Dynamical emergence cannot yet be brought within our analytical dynamics, because it represents a change in the currently fundamental dynamical form. In consequence, the laws of the emergent dynamics, whose form may depend on the particular constraint conditions, must then be separately described and applied. (This is so even where the occurrence of emergence is predictable.) Whence these

<sup>3</sup>See [Hooker, 1973; 1974]. It is presently obscure whether QM represents a 3<sup>rd</sup> alternative, see [Hooker, 1973; 1991].

<sup>4</sup>In recent times a major expression of this ideal was the Encyclopaedia of Unified Science, see [Neurath *et al.*, 1971].

<sup>5</sup>Just how much luck is needed is shown, e.g., by what is required to get astronomy, and hence Newtonian mechanics — our first sophisticated theory — started, see [Hooker, 1994].

laws and their generative conditions cannot be further compressed. (Moreover, at least all self-organisation involving a critical point process is evidently computationally impenetrable and so cannot be more than partially simulated either.) In short, limits to the reach of compression are substantially determined by the complexity of dynamics in relation to our mathematical representation tools. These limits bite at the same location as the ‘scandal’ of complex systems (see below). But it does not follow that they must be expressed solely or even primarily in terms of compression.

Reduction is concerned first with ontology (that is, with what exists), in particular with the ontological relationship between phenomena described in apparently different ways. What, for example, is the relation between clouds, snow, ice and rain, and water molecules in various dynamical arrangements (that is, under various initial and constraint conditions)? And, more challengingly, what is the relation between physiologically described function and biochemically described dynamical states and processes? The obvious response to make in each case is that the two are one and the same; that, for example, aerobic cellular respiration is nothing but ATP synthesis through glycolysis, Krebs cycling and electron transport. This is reduction by identification. The physiological function of respiration is identically reduced to, is identical to and so nothing other than, the dynamical system process of ATP synthesis through glycolysis, Krebs cycling and electron transport.<sup>6</sup> And this ontological relationship hinges on the dynamics involved.

Because relationships are expressed in language, there is the issue of how identifying descriptions of the putative two phenomena relate under reduction. The answer must ultimately be that they refer to the same thing (co-reference) but establishing the conditions for that is non-trivial and can be controversial. And the answer is bound up with that to the epistemic issue of what warrants affirming reduction. The answers to these issues, I suggest, must ultimately appeal to dynamical criteria, not only to purely logical criteria as usually presumed by philosophers.<sup>7</sup> Roughly, reduction obtains when the same one dynamics occurs and its affirmation is warranted when there is sufficient support for affirming same dynamics. It is then clear that the prevalence of condition-dependent dynamical laws [Hooker-a, b, this volume] means that no attempt to spell out these conditions in abstract generality will suffice, the identifications will always need to invoke the specific local systems conditions that determine the dynamics. Nagel’s general appeal to some kind of abstract deductive relationship ultimately remains, but only when it is clarified dynamically and refracted through the dynamical models;

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<sup>6</sup>All this assumes that the biochemical systems models involved are empirically supported and predictively and explanatorily adequate, an assumption made throughout this discussion. The issue of when and why that assumption is reasonable is again just the general issue of the nature of scientific method at large, see note 2. Also, this essay is concerned with empirical functions generally, not with the biological notion of proper function. The proper treatment of this latter notion is in fact given in terms of the notion of biological autonomy, see [Hooker-a, this volume, section 4.1.1].

<sup>7</sup>This was originally argued in [Hooker, 1981]; see further below.

it is too weak to stand by itself.<sup>8</sup> Moreover, only through appeal to dynamics can the relation between a new emergent existent and its constituting components be explained so as to coherently both give distinctive existence to the emergent entity and not compromise the fundamentalness of the components, especially for self-organised emergents (see below). And only in that way can the concomitant subtle entwining of emergence with reduction to yield a coherent naturalism be achieved (see below). This is why complex systems are so important to reduction and emergence: uniquely in them we find the subtle dynamical relationships that confront our formal efforts and drive us to improved understanding.

## 2 REDUCTION IN COMPLEX SYSTEMS: THE BASICS

There is a large philosophical literature on reduction in science, some of it proclaiming it and much arguing against it. The latter is especially prevalent in the domains of biology and other sciences concerned with internally complex system components where functionality (e.g. respiration) and its more intentional teleological forms is important to system integrity. Yet, from a scientific point of view it would be anomalous to claim anything less than a reduction, for example to claim instead just a correlation between the occurrence of functional and biochemical systems properties. Doing that would leave unexplained duplicate realities, one functional and the other dynamical. Against the advice of Occam's razor, it would leave two realms mirroring each other but running in parallel, for no reason more substantive than the different descriptive languages used, the one of functions (purely physical, but ultimately also including strategies, purposes, intentions and communication) and the other dynamical. Though among the debaters, in what follows I try to briefly summarise the state of philosophical debate from a commonsense scientist-friendly point of view, in order to focus on the underlying substantive issues at stake, especially those that concern complex systems.

*General objections.* Perhaps surprisingly, one group of philosophical objections to reduction in general argues that correlation must be accepted because identification is impossible. These arguments largely turn on semantic (meaning) considerations. To-be-reduced states are held to be characterised by novel properties that

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<sup>8</sup>Recently, Batterman tried to provide an alternative formal analysis of inter-theory reduction that was both more specifically tied to theoretical structure and yet still fully general by appealing to mathematical asymptotics, but this too ultimately fails because it parts company with the dynamics involved. See [Batterman, 2002] and for the critique see [Hooker, 2004]. Batterman appeals to the fact that a class of singular asymptotics shares the same formal structure, claiming that universality as the basis for nomic force. While not denying the mathematical facts, [Hooker, 2004] argues that this class covers nomically disparate cases, from changes in molecular interactions through those in non-interacting light wave structures to transforms of kinematical possibility structures, and it is hard to see what nomic basis these disparate cases could share in common. Batterman's position requires understanding the universality as a formal artifact and it is not clear how to present a nomic basis for it in those terms or what its ramifications might be elsewhere in physics. This remains an unresolved issue.

are not properties of the reducing substrate, e.g. macroscopic solidity and rigidity, much less colour, have been considered not properties of molecules, individually or collectively. Then the two sets of properties are held to have different meanings. And then it seems logically impossible to deduce the former from descriptions of the latter, as identificatory reduction requires. Again, talk of functioning, like respiring, the argument goes, has a very different meaning from talk of biochemical states and processes, so the two can never be identified, even if they are correlated.<sup>9</sup> The proper response to these kinds of objection is to point out, first, that they rely on *a priori* claims about the fundamentalness of semantics whereas what is known scientifically about language suggests that current semantics are better treated as themselves shifting dynamical emergents, not *a priori* constraints. In this spirit, second, there is an attractive alternative semantic basis to hand that obviates these problems, namely a same-dynamical-role criterion of property identity. This in turn supports a same-dynamics criterion of thing (object or process) identity and thus the identification of functions with their corresponding dynamical processes.<sup>10</sup>

Another group of arguments turn on the fact that the parallel mirroring is often not precise. Often there will be particular phenomenological conditions (for example, 'respiration') that do not nicely reduce to exactly corresponding underlying conditions (for example, ATP synthesis) of exactly the same scope. This is so because there are anaerobic organisms and various energy storage molecules, but it is also true because of complex dynamics. For instance, even Kepler's laws of planetary motion do not reduce exactly to a theorem of Newtonian mechanics, because planet-planet interactions produce small deviations from Kepler's generalizations. (This is the case under present conditions, but they may produce far larger deviations under other conditions, especially over long time periods.) Such complications will commonly arise wherever a more complex dynamics underlies more macroscopic/phenomenological observations. There are also cases where large mismatches occur. These are so large in the relationship of phlogiston chemistry to oxygen chemistry, e.g., that scientists deny that phlogiston exists even if its postulation served to codify a number of chemical relationships that survive the replacement. And there are intermediate cases, for example the imperfections

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<sup>9</sup>Conversely, Nagel once argued that if water is defined as  $H_2O$  then it is reducible, but not if it is not. See [Collier and Muller, 1998].

<sup>10</sup>This is argued in [Hooker, 1981, Part II]. What we know scientifically about language is that it is a recent evolutionary development, is characterised by rapid dynamical shifts in vocabulary, syntax and semantics as historical conditions change, and has a fundamentally action-centred intentional basis. In this light, a same-dynamical-role criterion of property identity is a far more plausible basis for semantics than so-called speaker intuitions; these have time and again been shown to simply project communal habit or vivid personal experience as cosmic truths. Scientists have had to learn the hard way that our concepts, even our seemingly most basic ones like hardness, simultaneity and consciousness, have to be continually reconstructed as we learn more because the world proves to be so deeply counter-intuitive. In consequence, scientists are wedded to constructing a unified dynamical conception of the world, not to naïve linguistic intuitions. Definitions are treated as works-in-progress and settling the 'right' ones is left until after a field matures.

of the thermodynamics-statistical mechanics relation.

In all these and other cases, the proper scientific response is that a warrant for reduction is ultimately concerned with establishing the capacity to systematically replace one kind of description with another kind that is equally or more precise, and equally or more predictively and explanatorily powerful when embedded in its scientific context. This satisfies the key cognitive aims of science. Reduction by identification forms one extreme of the reduction spectrum, where component ontology as well as relational structure is conserved under the replacement. The other extreme is occupied by cases like phlogiston where some significant relational structure, but not ontology, is conserved.<sup>11</sup> Labels are only useful to the extent they clarify, so in this case either of two labelling schemes is satisfactory: (a) label the entire continuum ‘replacement’ and retain ‘reduction’ for the identificatory extreme, or (b) retain ‘reduction’ for the entire replacement continuum, ‘identificatory reduction’ for its identificatory extreme and ‘replacement reduction’ for the opposite extreme. Either scheme locates the increasing discrepancy that characterises the relationship as one moves away from the identificatory extreme. This is what concerns scientists who frequently rely on the reduced theory because it is (typically) simpler and more immediately measurable, but who are concerned with understanding and managing the errors involved in doing so.

*Local objections.* These general issues aside, there are also various ‘local’ objections to reduction to consider. An important part of the philosophical objection to specifically biological reduction, for example, has really been to geneticism, to the idea that organisms could be reduced to just a collection of genes and gene-determined traits. Modern biology supports this objection, DNA is one biochemical component among many — if with a distinguishable role — and it is the dynamical system of all of them that is the reduction candidate for physiology. Again, various kinds of systems, e.g. those showing path dependencies, including systems whose parts play roles that depend on the specific system history, have been thought to raise difficulties for reduction because of their historical individuality. But it is easy to construct simple, clearly physical machines that also have those features, removing these objections.<sup>12</sup>

*Objections to functional reduction.* Setting aside such local objections as well, there remains only those objections that are specific to reduction of functions to systems dynamics. Here too there are various general objections of the semantic and mismatch kinds to deal with. One common semantic objection argues that if some property P (e.g. ‘pumps blood’) is defined as having the causal role or

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<sup>11</sup>Beyond that, sheer discontinuous replacement would occur, but it is hard to think of a substantial case in science where the replaced theory once had significant support (as opposed to simply being one of a number of initial hypotheses about some matter that were eliminated through experiment). For the replacement view see Part I of [Hooker, 1981] and, more informally, [Churchland, 1979]. Churchland’s elegant overall strategy, more subtle but powerful than it may appear, is itself explained in [Hooker, 2006].

<sup>12</sup>Any machine that assigns activity to parts as a function of their past total work hours and present system demand schedule will present this feature — see [Hooker, 1981, Part III], cf. e.g. [Miller *et al.*, 2000].

function of producing Bs (outputs) from As (inputs) then P cannot explain the producing of Bs from As. But why not? The fact is that an A-to-B transform occurs, hence the corresponding function is real and the explanation of A-to-B transformation appeals to the transform process, hence to the function. Indeed, it is better to avoid this (mis-)use of definition, replacing it by factual characterisation. In science, definitions are typically derivative and approximate, fallible and temporary, not basic (cf. Pierce), simply ready useful characterisations.<sup>13</sup> A further common objection is that our commonsense day-to-day function talk is rather imprecise for marrying up to dynamical systems specifications, e.g. compare ‘is boiling’ with all the specific ways fluids may convect. Another objection is that vague function descriptions can seem to cut across what turn out to be the dynamical process distinctions. For example, bird flight involves a lift function and a propulsion function, but the two functions cannot easily be separated into dynamically distinct components, as they can in contemporary aircraft. Objections of both these sorts can be resolved through a little careful analysis of language.<sup>14</sup>

There is also an inherent under-determination by any function, taken in isolation, of its correct embedding (that is, with which specific dynamical processes it is identical). While this has sometimes been taken as a fundamental objection to reduction, it ultimately reduces to a pragmatic issue of sufficient data. The problem is nicely illustrated in the case of the output of a network of electrical generators having a frequency variation less than that of any one generator. Some kind of feedback governing (i.e. regulatory) process is at work, but is it a real governor or simply the functional appearance of one at the network level? This latter is possible because connecting the electrical generators in parallel automatically creates a phase-stabilising mutual interaction among them without the need for a real governor.<sup>15</sup> This question is resolved by gathering other data about the network — this is the point of the unification criterion below.

*Conditions for function-to-dynamics reduction.* Setting these objections aside as well finally brings us directly to the substantive conditions for function to dynamical process reduction. And here a complex systems framework plays an important role.<sup>16</sup> For generality of application within complex systems, functions are considered under their most general aspect as maps carrying inputs to uniquely specified outputs, even if they are colloquially labelled as ‘pumps blood’ and the like. They

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<sup>13</sup>See note 10 and text. The metaphilosophical slogan here is: the bearable lightness of semantics. Since it is derivative and approximate, not basic, do philosophy by dynamics, not semantics, and let the semantics follow later, reflecting what has been dynamically established.

<sup>14</sup>See [Hooker, 1981, Part III] and, briefly, [Hooker 2004, Part V case I and case II end]. It would be possible to separate out static structural reduction — compositional reduction of one structure, say a rigid body, to another, say an atomic lattice — from functional reduction (cf. [Causey, 1977]). However, the overall issues turn out to be similar and most of the interesting cases are suppressed because they involve process-dependent structures, e.g. in the cell.

<sup>15</sup>For this example see [Dewan, 1976] and further [Hooker, 1981, Part III, pp. 508-511].

<sup>16</sup>This was recognized early by Hooker, see [1981], especially Part III, where the approach to reduction that follows was first developed. The construction there was intentionally cast in a complex systems context at a time when little philosophical attention was paid to this field (though not none, e.g. [Wimsatt, 1974] is an honourable exception).

are to be reductively identified with the dynamical processes that dynamically carry the (relevant, dynamically characterised) inputs to the (relevant, dynamically characterised) outputs, grouped together as the mechanisms that deliver the functionality (see 3 below). For example, cellular respiration, crudely globally specified, is the function that takes food and water as inputs and outputs stored energy and expired breath. Corresponding to this in the molecular description is a dynamical process — that is, a map carried by (biochemical) dynamical laws, constraints and initial conditions — that takes oxygen and glucose as inputs and yields ATP (and rarely, other forms of chemical energy storage) and carbon dioxide as outputs. Then the obvious requirement for identificational reduction is that the respiration functional map be embeddable into the corresponding biochemical process map without distortion (homomorphically embeddable). A further coherence condition is equally obvious: the collection of all such embedded dynamical maps, together with any non-functional data concerning the system, e.g. concerning its structure, should provide a single coherently unified biochemical cellular model that preserves or increases predictive and explanatory power.<sup>17</sup>

As the respiration example suggests, the dramatic example is that of cellular biology. On this score, Hooker [1981, Part III, pp. 515-517] considers the schematic reduction of Mendelian to molecular genetics from this point of view. Essentially, Mendelian genetics stands to molecular genetics as an input/output theory of some system (genes in, traits out) stands to a detailed internal dynamical theory of the system (DNA + cellular organisation in  $\rightarrow$  biosynthetic pathways + multi-cellular developmental dynamics  $\rightarrow$  spatially organised protein complexes out). This way of posing the relationship already shifts the focus from genes as isolatable objects to genes as functional units in complex system processes (cf. [Griffiths, Stotz, 2007]). Recalling the electrical governor example above, in the Mendelian case a sentence such as ‘ $X$  has a gene for characteristic  $A$  which is dominant’ would not be perspicuously analysable as ‘ $X$  has a component gene  $y$  that causes  $X$  to be  $A$  and  $y$  is dominant’, but instead as ‘There is some causal process  $P$  within  $X$  such that  $P$  causes  $X$  to be  $A$  under conditions  $C$  and  $X$  has  $P$  because of  $C$ ’, where  $C$  specifies the operative cellular constraints, including genome structure. This is a crucial shift, it corresponds to the end of genes-as-phenotype-determining-objects and the emergence of what is today the exploding fields of systems and synthetic biology, where the focus is on the complex regulatory mechanisms constituting the biosynthetic pathways. It is these that will determine whether and how the

<sup>17</sup>See [Hooker, 1981, Part III] and, briefly, [Hooker, 2004, Part V]. The basic reduction requirement, that functional maps are mirrored by dynamical maps, is in fact just the application of Nagel’s deductive reduction conception, rightly understood. Nagel [1961] shows how scientists arrive at reduction of a law  $L_2$  or property  $P_2$  of theory  $T_2$  respectively to a law  $L_1$  or property  $P_1$  of theory  $T_1$  by first showing how to choose conditions (real or idealised) under which it is possible to construct in  $T_1$  a law  $L_1$  or property  $P_1$  that will mirror (be a relevantly isomorphic dynamical image of) the dynamical behaviour of  $L_2$  or  $P_2$ . From that the reduction is shown possible through the identification of  $L_2$  or  $P_2$  with the mirroring  $L_1$  or  $P_1$ . Indeed, rather than having priority, the requisite ‘bridging’ conditions can be deduced from the mirroring condition, and then asserted as identities on the basis that doing so will achieve a reduction, supported in that light by claims of spatio-temporal coincidence or appeal to Occam’s razor.



cell will be recaptured as a complex dynamical system and the above coherence condition on successful reduction thus satisfied. As Hooker [1981, p.515] concluded two decades earlier: “The search for a reductive base for Mendelian genetics is now the search for the inner (in fact, molecular) mechanisms of genotypic-to-phenotypic production. . . . genes are not things but complexes of mechanisms.”

In that setting, and paraphrasing Hooker 1981, pp.515-516, what the reduction of Mendelian genetics to molecular genetics requires is that (1) the possibility, relative stability and conditions of change, of the cellular structures mediating the processes involved is explained by the basic biochemical laws, (2) as a result there is available a characterisation of the relevant initial and boundary conditions (other than structure) for any given system such that (3) the set of molecular mechanisms is unambiguously specified, (4) every true Mendelian sentence (in a suitably coevolved theory) has a corresponding condition realised within the complex of mechanisms and (5) for a specified Mendelian input/output relation and initial conditions a unique complex mechanism (biosynthetic pathway or complex of pathways) is selected such that (6) nomic Mendelian input/output relations (e.g. epistatic ones) are preserved and (7) the properties of the molecular model fit together in such a way that the ability to introduce Mendelian complex predicates such as ‘is dominant’ is explained, even though these predicates do not designate distinctive molecular properties at any level of structure. Though in 1981 they were just a gleam in the eye of molecular biologists, constructing such molecular models of complex mechanisms is what contemporary systems and synthetic biology are slowly making possible, aided by their high throughput experimental techniques. Despite its being in advance of the sequencing of genomes, constructing cellular models of its dynamical biosynthetic mechanisms is, as many biologists already knew in 1981 and Hooker reiterated, the really hard part of the reduction.

### 3 REDUCTION AND MECHANISM

This way of setting up the reduction criteria was designed to be appropriate for reduction within complex systems of the sort illustrated by respiration above, with reduction in simpler cases being simplified special cases. In fact, the reduction was explicitly designed to reduce functions to internal mechanisms as the specific kinds of processes that underlie functions (see [Hooker, 1981, Part III, p. 505]). Anticipating the later interest in mechanisms (note 19), mechanisms were understood there as law-like in their operation (“laws for the specific mechanisms”). And, as an explanatory requirement on their adequacy deriving from the further coherence condition on reduction (text to note 17), it was demanded that the operative mechanism laws should explain all of the empirical functional interrelations, e.g. dominance and epistatic relations among Mendelian genes (see [Hooker, 1981, Part III, p. 517]).

Following the example of respiration, specifying the mechanisms underlying functionality in complex systems can involve many different inputs and outputs

appearing at different stages of a process and at different levels of a multi-level system, sophisticated coordinated constraint relationships and multiple phase-shifted feedback/forward relations. A more familiar illustration is the reduction of automotive engine functions to complex organised engineering relationships among inputs, outputs and internal components from the chemical fuel, mechanical drive and electrical regulatory sub-systems. Such complex dynamical processes are essential if functions based on global organisation of the kind engines and organisms display are to be properly captured since no simple sequences of causal production relations can capture the global interrelations that constitute them. Capturing global constraints requires instead a complex network of dynamical interrelations with the requisite closure pathway structure (see [Hooker-a, this volume, section 3]). In these cases the active components in these networks can themselves be altered by their roles, even destroyed and re-constituted by them (in cells, not current engines). Thus mechanisms involving such irreducible constraints are not specifiable in terms of some fixed set of components, but rather in terms of their constituting dynamical processes. It is in this context that investigation of reduction within detailed models for specific systems are essential and valuable.<sup>18</sup>

As this discussion suggests, the embedding criterion essentially captures recent conceptions of a function to mechanism reduction, reducing both the cell and multi-cellular organisms to complexes of mechanisms.<sup>19</sup> Bechtel [2007] contrasts traditional universal law centred conceptions of explanation, reduction and generalisation with those appropriate to mechanisms and this may seem to challenge the present approach. He characterizes mechanistic explanation not in terms of logical inference but in terms of showing how a phenomenon is produced. Reduction is construed, not as a matter of deriving one set of laws from another, but as showing how parts and operations within a mechanism enable the whole mechanism to respond to conditions in its environment in specific ways. Finally, he characterizes generalization not in terms of universally quantified linguistic statements but in terms of similarities between model systems and other instances which share many of the same parts, operations, and modes of organisation, albeit often with some changes. (In many cases, the relations between systems are understood as features

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<sup>18</sup>See, e.g., [BooGERD *et al.*, 2005], cf. [Bruggeman, 2002; BooGERD *et al.*, 2007]. It is typical of physicists that they never deal with globally organised systems in their textbooks and hence tend to ignore global constraints and organisation, or assume that they can be reduced to separable momentary, local piece-wise interactions. If, for instance, the ambitions for condensed matter physics include direct modelling of the cell as simply a dynamical state like any other, as [Barham, 2004] supposes (cf. [Amaral and Ottino, 2004, sec. 3.2]), then they run directly into the issue of how to satisfy global organisational constraints, ones that are self-organised and adaptive at that. The challenge then is to understand how this project could possibly be carried out in a principled manner rather than ad hocly. Interestingly, Barham also doesn't see the problem. Moreover, he cites [Moreno and Ruiz-Mirazo, 1999] without noting that these researchers make such constraints central in the form of autonomy. Ironically for Barham, the autonomy global constraint is the foundation of natural value in that tradition — see [Hooker-a, this volume, section 4.1.1].

<sup>19</sup>On mechanisms see recently [Bechtel, 2007; Bechtel and Abrahamsen, 2005; Craver and Bechtel, 2006; Machamer *et al.*, 2000].

that have been conserved through processes of descent with modification.)

This is all appropriate, but once the condition-dependent character of laws is understood (see Hooker-a, b, this volume)], so that mechanisms are understood as possessing law-like operation, it will be appreciated that the above differences do not mark out a category opposed to nomic operation and explanation. Rather, they capture the entry of dynamical organization conditions into specification of dynamical process laws, in contrast with traditional laws that are taken to have minimal or no such conditions. Then the dynamical embedding approach provides a unifying basis for treating both kinds of laws as variants of one another, mechanisms having more complex, materially constrained dynamical processes that realise their functions (see section above): explanation appeals to the organisation of dynamical transform relations, reduction to the whole organised complex of dynamical processes and generalisation appeals to functionally relevant similarity of dynamical organisation.

In the general formulation above, Hooker [1981, Part III, pp. 503-505] identifies three kinds of descriptions of processes, the set of *mechanisms* (level  $L'_3$ ) constructed from the basic physical process dynamics (level  $L_3$ ), as the reducing mechanisms for the *functions* (Level  $L_1$ ).  $L_3$  is itself constructed piecewise from  $L_2$ , the basic *interaction/constraint dynamics* for the system. This is a very general approach to function and mechanism; there are as many functions and matching mechanisms as there are maps from any set of inputs to any set of outputs of the system. Let us then call these *basic* functions and mechanisms. While these provide a fully general basis for articulating function-to-mechanism reduction they are too indiscriminating to fully characterise productive scientific analysis of component mechanisms (cf. [Bechtel, Richardson, 1993; Bechtel, 2007]). A major challenge to mechanism theory posed by complex systems is to identify the explanatorily interesting mechanisms within the basic ones, that is, those mechanisms that realize the explanatorily important functions, whether of cells, cars, or cities.

Essentially this requires figuring out a principled basis for decomposing the total basic function of a system (the composition of all input/output maps for the system) into sub-functions. This is possible to the extent that the system is globally linear and locally modular, since then the total map can be written without loss of generality as a product of modular sub-maps. (The decomposition problem then recurs for each module.) But for non-modular systems, this is an important unsolved problem — and not just for philosophy, but for science as well.<sup>20</sup> Cellular biologists, control engineers and others concerned with complex systems would like to reduce the dimensionality of the system models and data they deal with and this again requires decomposing the total basic function in some principled way. But complex systems throw up organisational constraints that

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<sup>20</sup>Rosen (1991) calls decomposable systems *synthetic*, and those that are not *analytic*. He argues that the former are mechanical, but there is a clear sense in which analytic systems are not mechanical. His argument is somewhat obscure and contains both gaps and unsupported speculations, but in spirit it has much in common with the approach taken here.

simple linear mechanisms cannot cope with, especially feedback/forward phase-lag loop relationships and global organisational constraints like the requirement for the functional integration of automotive functions so as to simultaneously satisfy performance standards (acceleration and braking, driving control, ride quality, ...) or the equivalent global functional integration (autonomy) that defines life.<sup>21</sup>

According to contemporary analyses (note 19), mechanisms are characterised by four key features, operationality — the job they do (e.g. pump blood), componentiality — their component parts whose causal articulation structures them, causal regularity — their reliable operation, and organisation — the interrelatedness in their articulation. But global biological organisation challenges this overly ‘mechanical’ conception: components are often not stable but variously created and dissolved by the processes themselves and the globally coherent organisation this requires for overall persistence in turn requires a conception of globally coherent mechanisms. Mechanisms are conceived as organized processes, but a serious incorporation of organisation within them remains an outstanding issue.<sup>22</sup>

#### 4 SELF-ORGANISATION AND EMERGENCE

This issue also forms one facet of a serious outstanding problem posed by complex systems for a theory of reduction, viz. the occurrence of system levels. Following the dynamical approach to levels provided in [Hooker-a, this volume, section 3], levels proper are characterised by the presence of relatively macroscopic dynamical constraints. It follows that a level acts to ‘top down’ constrain component behaviour, in just the way that the iron lattice crystal does to create Fermi-band electrical current and lattice dissipative heat conduction. These constraints may be formed in many ways, e.g. both as a rigid crystalline structure emerges during the cooling of a liquid (iron bar) and as a liquid is heated to form Bénard convection cells. In the iron bar a macroscopic ionic lattice constrains the dynamics of microscopic component ions and electrons, constituting a distinctive dynamical level above them. In the Bénard convection case the macroscopic cell formation constrains the motions of all the molecular fluid constituents to same flow directions at all adjacent boundaries (cf. [Bishop, this volume, section 2.6]). Thus the formation of levels is directly linked to two other key complex systems features, self-organisation and emergence. Unravelling these issues will require introducing conditions under which reduction fails in a particular way. But it will then permit the account of reduction to be completed in an elegant, naturalistic manner, with the perhaps unexpected feature that irreducibility is a prerequisite condition for

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<sup>21</sup>On this notion of autonomy see references [Hooker-a, this volume, note 50].

<sup>22</sup>For some discussion of the issues posed for mechanism see Bechtel 2007 and especially Bechtel herein. In addition, the identification of the functions required for internal functioning is complicated by the role played by environmental features in generating organised behavioural patterns: ant and termite colonies, e.g., show complex organised behavioural patterns but these are largely environmentally generated, their individuals obeying only simple, myopic local ‘to-do’ rules.

coherent complex systems reducibility, the two conditions being intimately intertwined.<sup>23</sup>

In all systems it is true that the interacting components together create a dynamics that would not otherwise be present. When the outcome is surprising or unexpected or too complex to readily understand, scientists are apt to talk about emergent patterns.<sup>24</sup> When the outcome is more complicated or subtle behaviour, and the dynamics is entirely internal to the system, it is said to be *self*-organised.

There are many reasons why leaving things like that is unsatisfactory, among them that (i) no significant feature is addressed, our subjective surprise or sense of complicatedness and the like keeps shifting with experience, and (ii) these sort of criterion are dynamically so weak as to trivialise these ideas. But when it comes to strengthening the requirement, there is currently huge diversity of opinion about how both self-organisation and emergence are to be understood. Two broad approaches to identifying something more penetrating can be distinguished, the one epistemic and the other dynamical.

We are following the dynamical approach, but first consider the epistemic alternative. The epistemic approach tightens up the subjectivity by adding a clause along the lines that emergence or self-organisation occurs when the resulting system dynamics could not have been predicted from the known interaction rules of the components. This approach is attractive because there are many complex behavioural patterns that arise from the simplest interaction rules, for example with social insects (hives of bees and termite mounds), city traffic and even simple population dynamics as reflected in the logistic equation. However, it still ties the definition of evidently physical properties to a cognitive test. And if prediction is restricted to logical deduction from dynamics then almost everything self-organises since the demand for analytic closed-form solutions fails for all bar the simplest sets of differential equations.<sup>25</sup> So we pass to the option of a dynamical criterion.

Two dynamical distinctions stand out, and fixing on them avoids a long detour through a tortuous literature. The distinguished differences are those of (i) bifur-

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<sup>23</sup>This last was certainly unexpected in [Hooker, 1981], but not in [Collier, 1988; Collier and Muller 1998].

<sup>24</sup>An elegant way to systematise that talk has been provided by Ryan 2007, providing a powerful way to locate the phenomenon being referred to and identify the nature of claim being made about it, but it is largely agnostic about dynamical distinctions that might underlie it.

<sup>25</sup>As it stands, the text formulation is intolerably vague: Predicted by whom? Knowing what? Using what tools? And *prima facie* it makes an evidently ontological distinction (the existence of emergent behaviour) depend on a cognitive condition (human predictive capacity). If, in response, the criterion is instead formulated along the lines of 'cannot be derived from the set of interaction rules', or perhaps from just binary interaction rules, then these problems are lessened, but only to be replaced by the problem of what counts as an acceptable derivation. If derivation is restricted to logical deduction then almost everything self-organises since the demand for analytic closed-form solutions fails for almost all sets of differential equations. If derivation includes computational modelling of collective dynamics then almost all dynamics counts as derivable and nothing self-organises. Perhaps non-computable dynamics might be considered an exception, but since this occurs in quantum theory and other 'wave' dynamics as well as in critical-point bifurcations, it remains an insufficiently differentiated boundary. No satisfactory criterion of in-between scope is readily formulable.

cations, a structural instability leading to a shift in dynamical form, and (ii) the subset of bifurcations that lead to the establishment of a new system level. In the phase transition marking the formation of the iron bar, e.g., there is a bifurcation, but one in which a new level is formed. Specifically, a macroscopic pattern of inter-molecular relations is formed, the ionic crystal, which does thereafter have the power to constrain the movements of its molecular components through the formation of a new macro-scale force constituted in the ionic lattice bonds formed. Its formation alters not only individual component behaviour but also the specific dynamics under which they are now able to move: there are lattice vibrations and a Fermi conduction band in place of liquid molecular dynamics. That is, the phase change alters the force form of the dynamical equations that govern component behaviour. The new macro-scale force is able to retain the constraint relationship invariant under component fluctuations and exogenous perturbations, through lattice dissipation of these perturbing energies as sound and/or heat — that is, it is a top-down constraint.<sup>26</sup>

By contrast, the bifurcation in motion under gravity within a spherical bowl produced by raising a small mound in the bowl at some location introduces a shift in the spatial structure of the operative constraint force but no new macroscopic constraint. The same can be said of small alterations to avian flocking interactions and stigmergic rules for termite mound construction, both of which can nonetheless lead to shifts in collective behaviour. There is no dynamical constraint internal to either flocking birds or jamming motorists comparable to the iron crystal force that compels their members to wheel and turn just so, however complex their collective motion patterns. Finally, although from intersecting shallow waves on a gently undulating beach there emerges the most beautiful and intricate patterns, there is no comparable constraint of any sort formed by their interaction; shift the underlying sand structure and the dynamics can shift to entirely other patterns.

Between this last and the iron crystalline cases lie a variety of intermediate strengths of top-down constraint. For example, many social insect societies, like ants, are constrained by chemical reflexes and so exhibit some stronger constraint to their collective behaviours than bird flocking and jamming motorists, but it is still not comparable to the ferric crystal force. The crystalline cases are peculiar too in that their constituents are constrained in their total behaviour, whereas in other systems the top-down constraints can be expected to be more partial. For instance, the division of labour in bee hives seems much more collectively

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<sup>26</sup>The iron bar is a new macro-scale level with respect to its molecular constituents with its own characteristic dynamical interaction form. All other talk of levels either concerns measurement (liquid level), gravitation (level surface), or is metaphorical (semantic, social, abstraction, theory ... levels) and can thus be paraphrased away — or is confused. (The use of ‘level’ in Hooker’s  $L_i$  levels, 2 paragraphs above, pre-dates and violates this usage decision, referring only to descriptive ‘levels’.) Note that the presence of a top-down constraint does not fully determine the specific dynamical form of the system; both the virtual and real electrical governor arrangements (n.9 and text) exhibit the same phase-stabilising top-down constraint. Distinguishing between them is the electrical engineering ‘system identification’ problem — now with equivalents across all sciences, e.g. it is the problem of identifying correct cellular biosynthetic pathway models from fast-throughput experimental data that is a current central issue in systems and synthetic biology.

constrained than does their foraging patterns, which are more like bird flocking, even given their dancing. Cells in multi-cellular organisms seem as or more tightly bound, at least in respect of reproduction, specialisation and death (the 3 functions any viable community must control to some degree), and possibly in respect to other aspects of inter-cellular signalling.

It is natural to choose bifurcation as the dynamical criterion of emergence, for then a new behavioural pattern develops, and one whose occurrence is dynamically grounded in a shift in dynamical form. Bifurcations can take many different dynamical forms, but emergence is not concerned with the specifics of the process, only with something genuinely new coming into being from essentially the same resources. Bifurcation satisfies these requirements while providing the widest dynamically well-grounded criterion. In particular, this criterion applies independently of the computational character of the formation process and end state. While failure of computational penetrability represents a common alternative approach to characterising emergence, its status is problematic at this time.<sup>27</sup> It may be regarded as a positive feature of the bifurcation criterion that it does not depend on resolving the issues.

Thus defined, emergence corresponds to a part of the supra-component pattern formation spectrum running from level formation to non-bifurcational pattern shift. Calling the entire spectrum ‘emergence’ would reduce the notion to simply behavioural change, eviscerating its content. More informative to call the entire spectrum ‘pattern formation’. Confining ‘emergence’ to just the level-forming bifurcations would be to miss all those cases where the change is objectively grounded in a holistic change in dynamical form. This would leave these cases begging for a label that captures this sense of holistic transition. Thus neither of these is conducive to insight. Because the whole spectrum seems to have no other natural dynamical divisions, the intermediate usage — emergence = bifurcation — is ultimately most helpful.<sup>28</sup>

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<sup>27</sup>The predominant assumption is focused on unpredictability and thus linking irreducibility to some form of formal impenetrability, in particular the absence of a computational simulation of the dynamics, e.g. [Collier, 2008]). Bifurcations involving a critical point satisfy computational impenetrability, it seems, but they are not necessary for either a bifurcation or constraint-formation criterion to be met. Within present analytical mechanics the shift in dynamical form across constraint formation bifurcation is sufficient of itself to disrupt predictability. However its essence lies in the ontological reality of the internal constraint formation, not in unpredictability. This is fortunate since to pursue the impenetrability approach requires uncovering the details of the reach of compression into the domain of complex systems, and to distinguish in-principle limits from current pragmatic limits. But in fact there is relatively little known about this very difficult area. Beyond affirming vaguely that our present analytic techniques do not carry us very far, and that specific fragments of progress are occurring, see e.g. [Israel, Goldenfeld, 2006] among many, there seems little to be done but await mathematical developments.

<sup>28</sup>The criteria suggested in [Collier, 1988; Collier and Hooker, 1999; Hooker, 2004] is that of the more stringent requirement of self-organisation — see below. While this has the attraction of simplicity, since emergence = self-organisation, this now seems to me not to do justice to the differing orientations of the two concepts. Conveniently, there are two well-grounded dynamical distinctions available, quite well suited to the purpose, and it seems good sense to make use of them.

There is no physical mystery about emergence because the dynamics itself gives rise to the emergent dynamical form. The only components involved, e.g. forming any macro structures, continue to be the original dynamical entities from whose interactions the constraint emerged. From the component level perspective, this is simply local interaction placing mutual constraints on component behaviour in a way that eliminates certain collective possibilities (e.g. flowing as a liquid) while creating others (e.g. rigid collective interaction, refracting sound waves). But the formation of a new, holistic dynamical form makes it clear that the new collective possibilities are as real as were the old ones, just different from them (cf. [Bishop, this volume, section 2.6] on non-separable Hamiltonians).

There is some inclination to require that emergence means sufficient separation of the emerged level from its substrate that it has an intuitively clear separate existence. In consequence, the continuing presence and efficacy of the dynamical components is taken to mean that emergence has failed, e.g. that to some sufficiently god-like mind the iron bar is really just its components after all. But such imaginative vanities are irrelevant as well as ill-founded. What matters is that the iron crystal constraint is dynamically real, as determined by its energetic consequences, even though generated endogenously. The centrality of the dynamical criterion of new dynamical form is underlined by the fact that were the components to be fields and not particles, there would be no unique components available, yet the same dynamical criterion would demarcate emergence.<sup>29</sup> Whether or not the dynamical interactions that give rise to the emergent constraint change the components in the process, it is the fact that it is a *dynamical* bond that makes it possible to assert both the distinctive dynamical (if you wish: causal) character of the emergent entity and that it is comprised of (dynamical) components with the power to give rise to and sustain it. In this way emergence is naturalised for science.<sup>30</sup>

It is equally natural to choose level-forming bifurcation as the criterion of self-organisation (in the respects and to the degree it applies), for just this characterises the coming into being of a new self in the form of a new dynamical existent. The iron top-down constraint formation constitutes the coming into being of a new, individuated capacity to do work, expressed both endogenously in dissipation of perturbations and exogenously in rigid-body action. It is the arrival of a new dynamical individual characterised by a new dynamical form. The character of the new individual is constituted by its capacity to do new work, expressed in its dynamical form.

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<sup>29</sup>If there are unique, unchanging, spatio-temporally local, fundamental dynamical entities (for example, chemical ions as biochemical a-toms) then there is no emergence of new fundamental kinds, only existential dynamical emergents having these entities as ultimate components in various dynamical compounds. But top-down constraint formation of itself does not require this particular ontology. Fundamental non-linear fields would yield the same emergent result and there are no such local components, while mutant spatio-temporally local fundamental components would issue in fundamental kind emergence.

<sup>30</sup>This also ultimately invalidates Kim's treatment of diverse realisations of relatively macroscopic states and the conclusions he draws therefrom [Hooker, 2004, 5B; Wimsatt, 1994].



It will be observed that the same choice of terminology is at issue for ‘self-organisation’. It might be allowed that self-organisation should be more broadly defined to capture simply the central idea that the resulting pattern is brought about through the interactions of the system components. The colloquial term ‘organise’, as in ‘get organised’, encourages this wide connotation. But again, since the entire spectrum would then count as cases of self-organisation, this would reduce the notion to simply behavioural change, eviscerating its content. Alternatively, it might be argued that the term should be restricted to emergence in the sense of bifurcation, which remains weaker than that of top-down constraint formation. But this would include within self-organisation those systems whose bifurcation was produced by external manipulation, thus lacking an active ‘self’ in the process, and those systems that reduce their behavioural orderedness and/or complexity, thus lacking increased ‘organisation’ (even in the colloquial sense). While these outcomes are irrelevant to whether anything emerges, they still confuse any notion of self-organisation. Because the whole spectrum seems to have no other natural dynamical divisions, the narrow usage — self-organisation = level-forming bifurcation = level-forming emergence — is ultimately most helpful.<sup>31</sup>

This definition effectively replaces the idea of exhibiting more ordered and/or more complex behaviour as a characterising outcome of self-organisation by formation of a new level. For some this will be regarded as unsatisfactory. But at this time there does not seem to be any further dynamically well-grounded criterion that would do any better justice to this intuition. The obvious option to consider is adding some requirement concerning organisation proper (see [Hooker-a, this volume, section 3]). In this respect, it is worth noting that *self-organisation* need have little to do with organisation proper. This is as it should be. Organisation is a relational condition of systems where components play distinct roles but the roles are so interrelated as to produce a coherent global outcome. A simple illustration is found in the way the parts of a car engine are interrelated so as to deliver torque from fuel ignition; a profound example lies in intra-cellular organisation to produce a functioning organism. However, the end products of self-organisation need not involve the emergence of any organisation, as the case of crystallisation shows. Crystal formation is, rather, an instance of the formation of orderedness, rather than of organization. The unfortunately wide colloquial connotation of ‘organise’ conflates order and organization.<sup>32</sup>

With terminology settled we turn finally to the interrelationships between self-organised emergence and reduction in complex systems. A stable level is stabilised by damping or filtering component fluctuations, e.g. in the way that the iron crystal lattice dissipates perturbing fluctuations in lattice vibrations and the convecting cell dissipates them through molecular collisions. This insures that relatively

<sup>31</sup>The narrower usage is followed in [Collier, 1988; Collier and Hooker, 1999; Hooker, 2004].

<sup>32</sup>The conflation has a venerable history, e.g. while [Ashby, 1959] provides a constraint approach to what he calls organisation, what he defines is orderedness. The idea of global functional constraints, though implicitly available, had not come into focus and his formal approach excluded the dynamical bifurcations that underlie top-down constraint formation. For more on the order/organisation distinction see [Collier and Hooker, 1999], but cf. e.g. [Denbigh, 1975].

macroscopic properties have the stability we find them to have. This in turn making possible all the myriad processes that scaffold off them, e.g. the rigidity of iron components that make them fit to use in buildings and machines. And it is also on that general basis, and only on that basis, that we can track dynamical influences ‘up’ and ‘down’ through the component/supra-component levels, e.g. track the consequences of an ionisation event in a Geiger counter through to the output click [Hooker, 2004, Part 5B]. By providing higher level structure for well characterised lower level processes, macroscopic constraints underpin the reduction of the functions served to their dynamical process mechanisms. (And of course the constraints themselves and attendant structures reduce to dynamical compounds of the components whose interactions constitute them.). That is, it is the very stability of the emergent constraints that both make possible well-defined functions like the Geiger counter’s detection and also the well-defined dynamical processes underlying them (e.g. ionisation and amplification in the Geiger counter) and in doing so provide the basis for functional reduction [Hooker, 2004, section 5B]. On the other hand, these macroscopic constraints herald the presence of a new dynamical existent created by an over-arching dynamical bond, so the new dynamical existent precisely cannot be reduced to components alone. It is an irreducible new being.

Thus, contrary to the standard view of reduction and emergence where they are opposed, this discussion shows that emergence underpins functional reduction, and reduction, both compositional and functional, in turn allows specification of the processes and conditions (initial and boundary) that underpin emergence. The two are thus intricately interwoven and mutually supportive.

## 5 CONDITIONS FOR IRREDUCIBLE EMERGENCE

Significantly, self-organisation is a process where dynamical form is no longer invariant across dynamical states but is rather a (mathematical) function of them. Despite advances in non-linear mathematics and computing, this remains an ill-understood process.<sup>33</sup> These circumstances set the scene for examining, first, what a minimalist logical account offered by philosophers might do to clarify emergence (section 5.1) and, second, what can be said about its dynamical conditions in relation to the foundations of classical dynamics (section 5.2).

### 5.1 *Logical conditions for irreducible emergence: supervenience*

Within contemporary philosophy the notion of supervenience is the dominant, and minimalist, approach to characterising emergence. *A* states are supervenient on states *B* if and only if every change in *A* requires a change in *B* (though not necessarily vice versa). For the intuitions driving this formulation, consider a standard macro-micro relationship: The iron bar is supervenient on its molecules, no

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<sup>33</sup>For the spread of non-linear research see e.g. <http://arXiv.org/archive/nlin/>; Ma and Wang [2005] provides a mathematical introduction to bifurcations more generally.

properties of the macroscopic bar ( $A$  states) can change without the change being dynamically grounded in appropriate molecular changes ( $B$  states), though there will be many molecular changes that produce no macroscopic change. Supervenience specifies a many-one map (a mathematical function) from  $B$  to  $A$  (many different arrangements of iron molecules can produce the same bar). Thus  $A$  states correspond to sets of  $A$ -equivalent  $B$  states and  $A$  state transitions (say from  $A$  to  $A'$ ) correspond to sets of  $B$  transitions (all those allowed from any member of the  $B(A)$  set to any member of the  $B(A')$  set).

The supervenience relation is minimalist because nothing further is said about how the  $B - A$  relationship works. There is no dynamical account of the  $B - A$  relation specified, as there is for the iron bar, e.g., where we know which molecular changes are required to dynamically explain which macroscopic changes. Indeed, and quite unlike the iron bar, supervenience is strictly compatible with the  $A$  states being causally unconnected to the  $B$  states, instead just 'running in parallel'. (The  $A$  states might, e.g., be mental states in some spiritual 'ether' and the  $B$  states brain states.) Supervenience is also compatible with the  $B - A$  mapping itself being a function of other variables, e.g. of time, though most commentators assume not. The desire to accommodate the parallelism option, together with philosophers favouring use of purely formal relations, may explain the minimalism. In any event, it means that we cannot hope to derive much enlightenment from applying the supervenience concept. The only interesting consequence of applying it is that fixing the  $B$  states (and the values of any other relevant variables) fixes the  $A$  phenomena. This expresses the degree of 'bottom-up' determinism built into supervenience.

It also makes the  $A$  states look parasitic on the  $B$  states, since the  $A$  level can't change without the  $B$  states changing. This can be taken to imply  $B$  causal leadership, and conclude that the  $A$  level has no causal powers other than those possessed by its  $B$  substrate. (The aggregate is an especially plausible example here.) An obvious next move is the application of Ockham's razor to remove  $A$  states as separate existents, through reduction to the  $B$  states. This would have supervenience support reduction and exclude emergence.<sup>34</sup> But this need not be so. As just noted, supervenience strictly says nothing about the actual physical relation of  $B$  states to  $A$  states, so it is quite compatible with supervenience that that relationship is, e.g., one of dynamical self-organisation. In that case there are dynamically based  $A$  causal powers that can constrain  $B$  dynamics and thus  $A$  states are irreducible to  $B$  states. There is no contradiction here, simply that supervenience is in fact compatible with any of the relationships, illustrated above, ranging from aggregation to self-organisation. This reveals the discriminatory poverty of the supervenience relation.

Dynamical analysis provides a much richer language in which to discuss the

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<sup>34</sup>Kim takes  $B$  as the physical states and assumes determinism, to obtain this version of supervenience [Kim, 1978]: If all of the (determinate) physical facts are determined, then all (determinate) facts are determined. Kim supposes reduction to follow, but things are not so simple (see following text).

possibilities, distinguishing all the relationships across the range from aggregation to self-organisation. In particular, for self-organisation the dynamics itself shows how the (relatively) macro level constraint is determined by the states of its micro constituents and so is supervenient on them, yet can nonetheless also constitute a constraint on them. Here dynamics gives the constraint a subtle status that eludes conventional formal analysis, and supervenience. Understanding is only obtained when supervenience is replaced with (or perhaps enriched with) dynamical relations.

Thus dynamical determination, = there being only one dynamical possibility for the collective dynamical state/property, cannot be equated with logical determination, = the collective dynamical state/property is logically derivable from, can be expressed as a logical sum of, its constituent states/properties. The latter includes only the weaker, aggregation and simple pattern formation relations, while the former also includes the stronger bifurcational, especially self-organisation, relations where deduction fails because, as noted earlier, there is no single mathematical framework within which dynamical form shift is a well defined transformation.<sup>35</sup> Thus reduction fails in these cases but holds for pattern formation and aggregation cases.

## 5.2 *Dynamical conditions for emergence*

The ambitious aim here would be to fully characterise the situation of complex systems dynamics in relation to dynamical theory, but this is a task far too deep and complex, and unfinished and rapidly evolving, to contemplate. So the scope of the discussion is restricted to just classical dynamics (dynamics of classical mechanics) and the aim is restricted to characterizing the relation of some key general aspects of classical dynamics to some key general aspects of complex systems dynamics.

The classical Newtonian laws of motion constitute a very general dynamical framework for particle mechanics that in itself places no restrictions on the kinds of material systems involved, e.g. whether involving charged particles or dissipative forces like friction. It is always possible to attempt to mathematically analyse such systems from first principles in order to discover their behaviour, however success in this is not guaranteed and is in fact typically very hard or impossible beyond the simpler classes of dynamics, e.g. for systems undergoing phase changes (e.g. gas to liquid to solid).<sup>36</sup> However, over the past 250 years a generalised analytical framework has been constructed for classical dynamical analysis — the Lagrangian/Hamiltonian formalism — that directly or approximately covers a wide range of cases and serves as the core of analytical classical dynamics.<sup>37</sup> Moreover the underpinning principles have the further merit of providing the

<sup>35</sup>Indeed, these trajectories may be computationally strongly inaccessible, for example through all critical point phase transitions, and so certainly cannot be deduced from micro information.

<sup>36</sup>Cf. Humphries on the mathematical tractability of computational templates, section 3.5 of [Humphries, 2004] and the discussion in [Strogatz, 1994, p.11] and text.

<sup>37</sup>For a nice introduction see [Butterfield, 2004a; 2004b]. Classic texts here are [Goldstein 1950; Arnold, 1978]. Butterfield [2004a, note 6] offers a brief guide to others. [Bloch, 2003]

means to also treat continua and force fields, especially the electromagnetic field, and of furnishing principles for subsequently generalising mechanics to relativistic and quantum formalisms. Thus this analytical framework has not only come to constitute “classical mechanics”, its core exemplifying our conception of purely mechanical systems, but it has come to constitute the foundational core of all general dynamics, the focus for attempts to understand relativistic and quantum theories and their union.

In this setting it may be thought that insofar as complex systems lie outside this core, as most of them do, it is simply because solving their dynamics is still too hard as yet and we are driven to piecemeal tricks, model by model. We *are* at present largely driven to piecemeal tricks, and may or may not be stuck there, but there is more at stake than progress in practical mathematical methods. Complex systems characteristically centre on dynamical conditions at variance with the assumptions underlying the classical core and thus face us with a fundamental dilemma, and a small scandal.

To understand what is involved, begin by noting that a classical mechanical system consists of material components (the system elements), interactive forces among these components which provide the intrinsic forces, and external constraints or boundary conditions placed on the system (e.g. for a gas or fluid, the walls of its container) which provide the extrinsic forces acting on it. In the absence of constraints there are only intrinsic forces and a Newtonian dynamics can be formulated with mutually independent dynamical variables expressing the intrinsic degrees of (dynamical) freedom of the dynamics. External constraints are typically represented as restricting the movements of a system (e.g. gas container walls) and are assumed to apply sufficient forces to achieve this. While the non-linearity of the intrinsic forces is the primary means through which dynamical complexity is produced, it is the character of the external constraint forces that is the key factor in the formation of an analytically tractable treatment of the dynamics. External constraints typically enter unknown forces into the dynamics, so that a determinate Newtonian dynamics cannot be specified, and they result in interdependencies among the intrinsic dynamical variables that have to be accommodated, so that an unambiguous representation of the dynamical possibilities cannot be formulated.

The standard dynamical formalisms for mechanics are those of the Lagrangian and Hamiltonian forms; here just the more general Lagrange formalism will be discussed. To construct a Lagrangian model for dynamics it is necessary to restrict consideration to those systems where the external constraint forces act orthogonally to all allowed system motions<sup>38</sup>, so that the system does no work against external constraints (constraint force orthogonality). This defines a ‘constraint (hyper) surface’ in the system configuration space to which the constraint forces

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offers an introduction to the treatment of D’Alembertian but non-holonomic constraints, see also [Flannery, 2005].

<sup>38</sup>These are the ‘virtual’ displacements of a system, as opposed to actual displacements over some small time interval occurring under the influence of the intrinsic forces as well.

are everywhere perpendicular (orthogonal). The dynamics on this surface is thus effectively separated from the external constraints, each is unaltered by the other throughout the system motions — this is expressed in D'Alembert's principle. The dynamics is then open to a purely intrinsic characterisation. It is, however, non-trivial to make good on this promise since the issues of interdependencies among variables and unknown external constraint forces remain unresolved.

If in addition the external constraints are holonomic — literally: express a whole or single law<sup>39</sup> — then the system dynamics may be re-formulated on their D'Alembertian constraint surface in terms of new generalised coordinate variables that are mutually independent. The dynamics now has the form of a free (unconstrained) system. The effect of the constraints, implicit in the geometry of the constraint surface, is now also implicit in the construction of the new variables for intrinsic motion on it. Lagrange equations of motion can then be formulated for the system. This resolves the variable interdependency problem introduced by constraints. We think of these systems as simply following a least-action path in a pre-specified purely geometric framework and hence as distinctively 'mechanical' in nature.<sup>40</sup> Further, the method of Lagrange multipliers permits solving the system dynamics on the constraint surface (that is, specifying the action geodesics) without knowing the external constraint forces. Rather, once the dynamics is known, the external constraint forces can be reconstructed as the forces they need to be to maintain the external constraints during the system motion. This resolves the problem of their being initially unknown.

More could easily be added. Theoretically, e.g., the Lagrangian form represents a simpler set of equations to be solved than is Newton's and the Hamiltonian formulation extends this trend. Practically, e.g., the familiar form of the Lagrangian as kinetic minus potential energy can be derived if the forces can be expressed as the gradient of a single function. Moreover, the Lagrange multiplier method extends to some classes of non-holonomic constraints as well (specifically semiholonomic and exact linear constraints [Flannery, 2005]) and there may be progress with others (e.g. cf. [Fernandez, Bloch, 2008; Krupková, 2009]). However, the

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<sup>39</sup>Holonomic constraints may be written as some function of the space-time geometry in which the system moves. Specifically, they satisfy an equation of the form  $f(r_1, r_2, \dots, r_n, t) = 0$ , where the  $r_i$  are system coordinates and  $t$  is time. This expresses the effect of the constraint forces while not specifying the forces themselves. (The forces are often known only after the main problem is solved.) While smooth (frictionless) sliding under gravity on a sloping plane is a case of holonomic constraint, a spherical bead rolling smoothly on the outside of a cylinder is not because the constraint alters its basic character when the bead falls off. Essentially, for the constraints to be holonomic means that they may be expressed purely geometrically, so that they are independent of the behaviour of the system. Independence fails in the case of the bead on the cylinder, there is a change of constraints at a space-time location determined by the bead's motion. (Note that the reverse relation does not hold, e.g. though independent of system behaviour, containment walls do not form holonomic constraints.)

<sup>40</sup>Especially if neither the external constraints nor the potential energy are time-dependent, the usual textbook case. But note that the intrinsic force potential is still produced by the system components themselves and any internal constraints will form in virtue of such forces; we need to be wary of claims that there is any sharp gulf separating mechanical systems from the more complex non-mechanical ones to which we shall shortly point.

general result above states the nub of the relevant matter here. It suffices only to add that D'Alembert's principle introduces the first of the variational formulations of dynamics whose extension stands at the core of generalizing dynamics to encompass the relativistic and quantum domains. These variational principles are considered to lie at the core of analytical dynamics.<sup>41</sup>

Nonetheless, for many complex systems the external constraints that apply depend on what the dynamical state is, so that constraint holonomicity fails, blocking the path to further re-construction of the dynamics. This would, it seems, be true for all globally constrained, functionally resilient (often called robust) systems that can adapt their process organisation to compensate for damage or other altered circumstances, as can living cells. Moreover, many systems where constraints are a function of state also do work on the constraints, physically altering them over time. Examples include (i) a river altering its own banks, an accumulative process where the current constraints (banks) are a function of the history of past flows (currents), (ii) intra-cellular biochemical reaction processes where molecular structures constraining some processes are the products of other processes and vice versa; and (iii) any self-organisation where the constraint formed becomes an external constraint for subsequent processes (Bénard cell and iron bar formation, etc.). In all these systems constraint orthogonality fails. With this failure the most basic precondition for achieving the core analytic construction fails. There is then no general analytical mathematical formalism available for dynamical behaviour. Moreover, most of these systems have proven recalcitrantly impenetrable to analysis and essentially each system has to be treated individually on its merits. There are stronger claims, e.g. that all chemical reactions are irreversible thermodynamic processes defying analytical dynamical characterisation [Prigogine, 2003], but those above suffice. It is among these that we find all systems exhibiting emergence and organised global constraints and many other of the characteristic features of complexity (see [Hooker-a, this volume, section 3]).

There is then a hiatus between those systems whose dynamical foundations we think we understand (Lagrangian systems) and those systems that manifest the features characteristic of complexity. D'Alembert's principle fails for the latter systems, undermining the applicability of the very variational apparatus that we take to underlie all fundamental dynamics. In this way, complex systems challenge the reach of our deepest analytical understanding of dynamics and thus present a fundamental dilemma about how to approach dynamics: retain the present approach and exclude complex systems or search for some new, more generous foundations for dynamics. There is also a whiff of scandal here as well, namely, the unfortunately increasingly common scandal of dynamics textbooks simply ignoring these deep problems. This is especially scandalous at a time when, as this

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<sup>41</sup>In this process analogous conditions to those necessary for the representations (no virtual work, holonomicity) are assumed universally applicable, plus, for the Hamiltonian formulation, also the assumption of the mutual independence of coordinates and momenta [Goldstein, 1950, p.227]. This is a pivotal point for reflection on how Newtonian systems that violate these requirements, as emergent systems will do (below), could be represented in relativistic and quantum terms.

volume demonstrates, complex systems are having such a huge impact on science, including on the mathematical techniques for analysing dynamical systems. More important, however, is scandalous question-begging in favour of the existing approach by the commonest textbook response, which implies that there is only a pragmatic issue of mathematical resources involved.

The ground offered for the latter is that ultimately all systems, constraints as well as components, can be represented at the fundamental component level (however components themselves are represented). Thus all external constraints are then represented as forces deriving from yet further fundamental components. The gas container becomes a metallic lattice plus free electrons, and so on. These external components may then be added to those of the systems they constrain to form a dynamics that is constraint-free (constraint forces = 0) and hence Lagrangian methods suffice.<sup>42</sup> If we cannot solve these systems then it is simply because there are too many components involved, a pragmatic rather than a principled difficulty.

It is certainly the case that constraints can be shifted between external and internal status. Consider the iron bar again; its ion lattice formed as an internal constraint but, once formed it may be treated as an external constraint for lattice processes such as sound and heat propagation and Fermi band electrical conduction. However should the energy in these processes become sufficient to perturb the lattice ions sufficiently to do work on the lattice, then the lattice has to be again brought within the dynamics as an internal constraint. The argument is that ultimately this is true of all constraints.

About this argument, the following points are germane to estimating its persuasiveness. (1) This is at best an in-principle argument, a necessary condition for its coherence being a proof that the actions of the fundamental forces always permit definition of a suitable D'Alembertian surface. (I assume that the method of Lagrange multipliers then works.<sup>43</sup>) Given that many of these systems do real work

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<sup>42</sup>See e.g. [Goldstein, 1950, p.14] for this view.

<sup>43</sup>Flannery [2005] emphasises the compatibility of the method of Lagrange multipliers with the D'Alembertian surface (i.e. its ability to identify a field of extremal flows on the D'Alembertian surface) as the key condition for the coherent formulation of a Lagrangian dynamics. Others (e.g. Butterfield) emphasise obtaining a D'Alembertian surface at all as the key condition, with the multiplier method treated only as a useful add-on. Lagrange multiplier methods are not often discussed in dynamical contexts. One immediate issue is whether these two conditions are in fact equivalent — does it follow that if a D'Alembertian surface is definable then the method of Lagrange multipliers works for it and, conversely, if the method of Lagrange multipliers is well defined then a corresponding D'Alembertian surface is defined? This has some interest in the light of the extensions of Lagrangian formulation to various classes of non-holonomic constraints, e.g. [Fernandez, Bloch, 2008; Krupková, 2009]. Another issue is whether there is a general characterisation of the classes of dynamical systems for which the D'Alembertian variational principle yields a minimum and for which it yields a maximum (the method itself only requires an extremal value), and a general explanation of these differences. Finally, there is the less defined issues of (a) how to insightfully characterise the respective contributions of the intrinsic dynamical non-linearities and the constraint structure to the generation of system dynamics that are analytically untreatable and (b) whether there is a general characterisation of the class of non-holonomically constrained systems that are treatable with an extension of Lagrangian methods



on their constraints, it is not obvious how this proof would succeed. Until this proof is available, it remains a fact that none of the constrained non-D'Alembertian systems have a coherent Lagrangian mechanics specified. (2) Nor can any system enlargement detract from the dynamical reality of constraint formation. (It certainly cannot be argued that internal constraint formation is only a convenient illusion, since its reality is attested by the energetic difference it makes, expressed as a difference in the system work function.) In this light it is difficult to see how systems showing self-organised emergence could be reduced to presenting merely pragmatic barriers to knowledge of solutions. To take this latter view requires presuming that in the fundamental representation all top-down constraint formation becomes representable as a process within Lagrangian dynamics. Since precisely in such processes the system changes dynamical form, hence would change Lagrangian form, it is unclear how the Lagrangian apparatus could accommodate that requirement.<sup>44</sup> Thus the response begs the question against these arguments, without providing a demonstration that there is a real solution available.<sup>45</sup> The basic dilemma persists.

## 6 CONCLUSION

The logic machine vision is not dead. Condition-dependent laws still compress and dynamical equation sets still provide implicit compressed representations even when most of that information is not explicitly available without decompression. And, paradoxically, there is still the determined march of fundamental analytical dynamics expanding its compression reach toward a Theory of Everything - even while the more rapidly expanding domain of complex systems dynamics confronts its assumptions and its monolithicity. Nor does science fall apart into a dis-unified aggregate of particular cases since, with fundamental dynamics as a backbone,

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(other than that they satisfy your requirement of the compatibility of the method of Lagrange multipliers with the D'Alembertian surface).

<sup>44</sup>It seems true that if a universal D'Alembertian, holonomic model is to apply then all the complex phenomena of complex systems, such as emergence, must be explicable in terms of dynamically valid coarse graining on the underlying fundamental dynamics. However, coarse graining in itself may not help much in developing a coherent mathematical account simply because its point, reflected in renormalization, is to avoid appeal to the dynamical details. Further, in those cases of singular asymptotics of bifurcation, it is hard to see how coarse graining can provide intelligible legitimacy to an underlying analytic model since that model would seem to require ultimate continuity of change, in contrast with the discontinuous change involved in singular asymptotics, not to mention also requiring motion solely within some single dynamical form.

<sup>45</sup>It also requires presuming that in the fundamental representation all global organisational constraints, such as those of autonomy, also become analytically representable. But again, the Lagrangian formalism has no apparatus for representing such constraints. To insist that they reduce to just pair-wise interactions among fundamental components is to again beg the question of their dynamical reality (their contribution to dynamical form). While it is not clear just how such constraints are to be represented, it seems clear that they must be more palpable than mere pair-wise correlations — think in this respect, of the imposed global coordination of Bénard convection cell circulation..

complex matching up of models across theoretical and empirical domains then articulates its model-structured skeleton. Here is included the delicately entwined dance of emergence and reduction providing constraints on compression that also permit its expansion. However, while the vision is not dead, it is currently substantially more complexly structured through model similarities and differences than that initially envisaged and we are left with deep questions about compression unresolved.

## ACKNOWLEDGEMENTS

I am indebted to a discussion with Paul Humphries and to several with Gordon McLelland that clarified several key points.

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